Math 53, Discussion 107

Thursday, March 19, 2020 5:51 PM

1) Evaluate $\iint_D x(y-1)\,dA$ where D is region bounded by $y = 1 - x^2$ and $y = x^2 - 3$, answer should be 0

2) Evaluate $\iint_D \sqrt{1+4x^2+4y^2}\,dA$ where D is the bottom half of $x^2+y^2=16$, answer: $\frac{1}{12}\pi\left(65^{\frac{3}{2}}-1\right)$

3) Evaluate $\iiint_E 6z^2\,dV$ where E is the region below $4x+y+2z=10$ in the first octant. Answer: $\frac{625}{2}$

1) $\displaystyle\int_{-2}^{2}\int_{x^2-3}^{1-x^2} x(y-1)\,dy\,dx$

$1-x^2 = x^2-3$

$4 = 2x^2$

$x = 2, -2$

$\int xy - x\,dy$

$\left.\frac{xy^2}{2} - xy\right|_{y=x^2-3}^{y=1-x^2} = \frac{x(1-x^2)^2}{2} - x(1-x^2) - \frac{x(x^2-3)^2}{2} + x(x^2-3)$

$= \frac{1}{2}\left(x\left(1-2x^2+x^4\right)\right) - x + x^3 - \frac{1}{2}\left(x\left(x^4-6x^2+9\right)\right) + x^3 - 3x$

$x\left(\frac{1}{2}+9-1-3\right) + x^3\left(-1+1+1-6\right)$

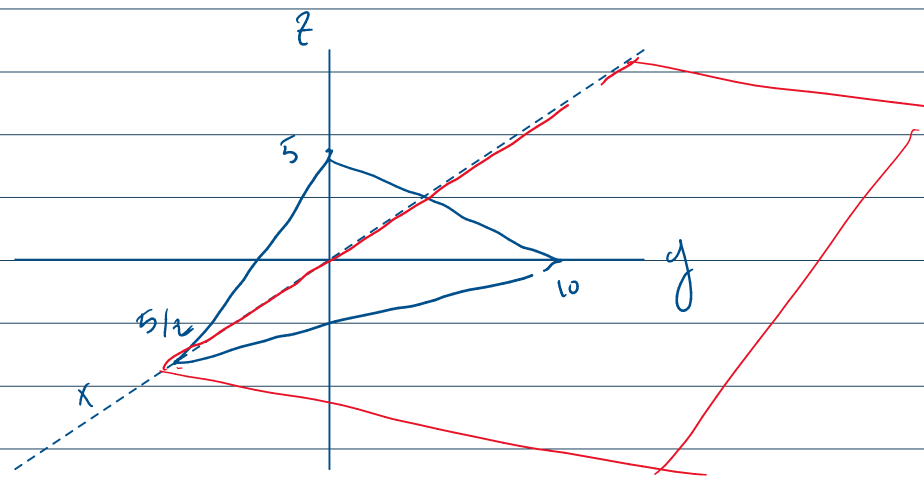
2) $\displaystyle\iint_D \sqrt{1+4x^2+4y^2}\,dA$ $D =$ bottom half of $x^2+y^2=16$

2π 4

$$\int_{\pi}^{2\pi}\int_{0}^{4} \sqrt{1+4r^2}\; r \; dr\, d\theta$$

Let $u = 1+4r^2$ $\quad \frac{du}{dr} = 8r$ $\quad dr = \frac{du}{8r}$

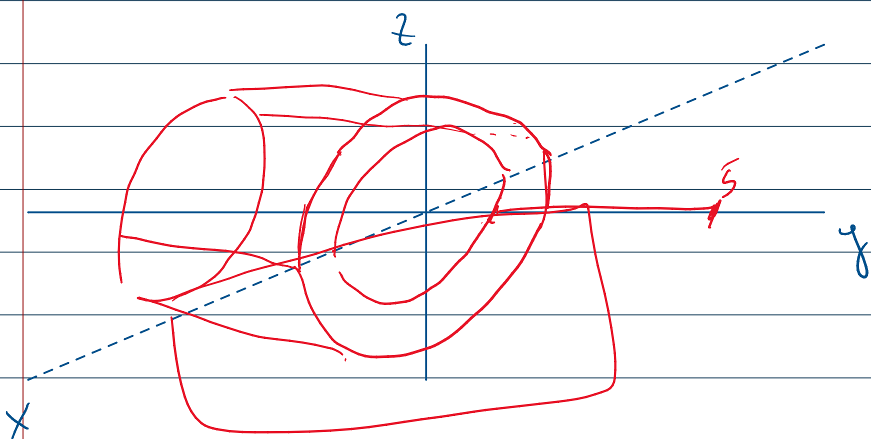
3) $$\int_{0}^{\frac{5}{2}}\int_{0}^{10-4x}\int_{0}^{(10-4x-y)\frac{1}{2}} 6z^2 \; dz \; dy \; dx$$ $\qquad 4x+y+2z=10$



1. Evaluate $\iiint_E e^{-x^2-z^2}\,dV$ with E the region between the cylinders $x^2+z^2=4$ and $x^2+z^2=9$ with $1 \le y \le 5$ and $z \le 0$, answer: $2\pi(e^{-4}-e^{-9})$
2. Compute $\iiint_E 4xy\,dV$ with E the region bounded by $z = 2x^2 + 2y^2 - 7$, and $z = 1$
3. Compute $\int_{-\infty}^{\infty} e^{-x^2}dx$ (hint: compute $\left(\int_{-\infty}^{\infty} e^{-x^2}dx\right)^2$ using polar coordinates)

1) Want y to be height in cylindrical coordinates. Let $x = r\cos\theta$, $y = y$, $z = r\sin\theta$

$$\int_2^3 \int_{-\pi}^{0} \int_1^5 e^{-r^2}\, r \quad y\, d\theta\, dr$$



$$\int_1^2 x\, dx = \frac{x^2}{2}\Big|_1^2 = \frac{4}{2} - \frac{1}{2}$$

$$\int_1^2 y\, dy = \frac{y^2}{2}\Big|_1^2 = \frac{4}{2} - \frac{1}{2}$$

$$\int_1^2 \cancel{\theta}\, d\cancel{\theta}$$

Let $I = \int_{-\infty}^{\infty} e^{-x^2}\, dx$

$$I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2}\, dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2}\, dy\right)$$

Shaky:

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2} e^{-y^2}\, dx\, dy$$

$$= \iint_{\mathbb{R}^2} e^{-x^2-y^2}\, dx\, dy = \int_0^{2\pi}\int_0^{\infty} e^{-r^2}\, r\, dr\, d\theta$$

$$= 2\pi \int_0^{\infty} e^{-r^2}\, r\, dr$$

$$= 2\pi \int_0^{\infty} e^{-r^2} r\,dr$$

$$= 2\pi \left(\frac{-1}{2} e^{-r^2} \Big|_0^{\infty} \right]$$

$$= -\pi \left(e^{-\infty} - e^{0} \right]$$

$$I^2 = -\pi (0 - 1) = \pi$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} dx = 1$$